# Behavioral economics

Lecture 7 - **Analytical game theory**

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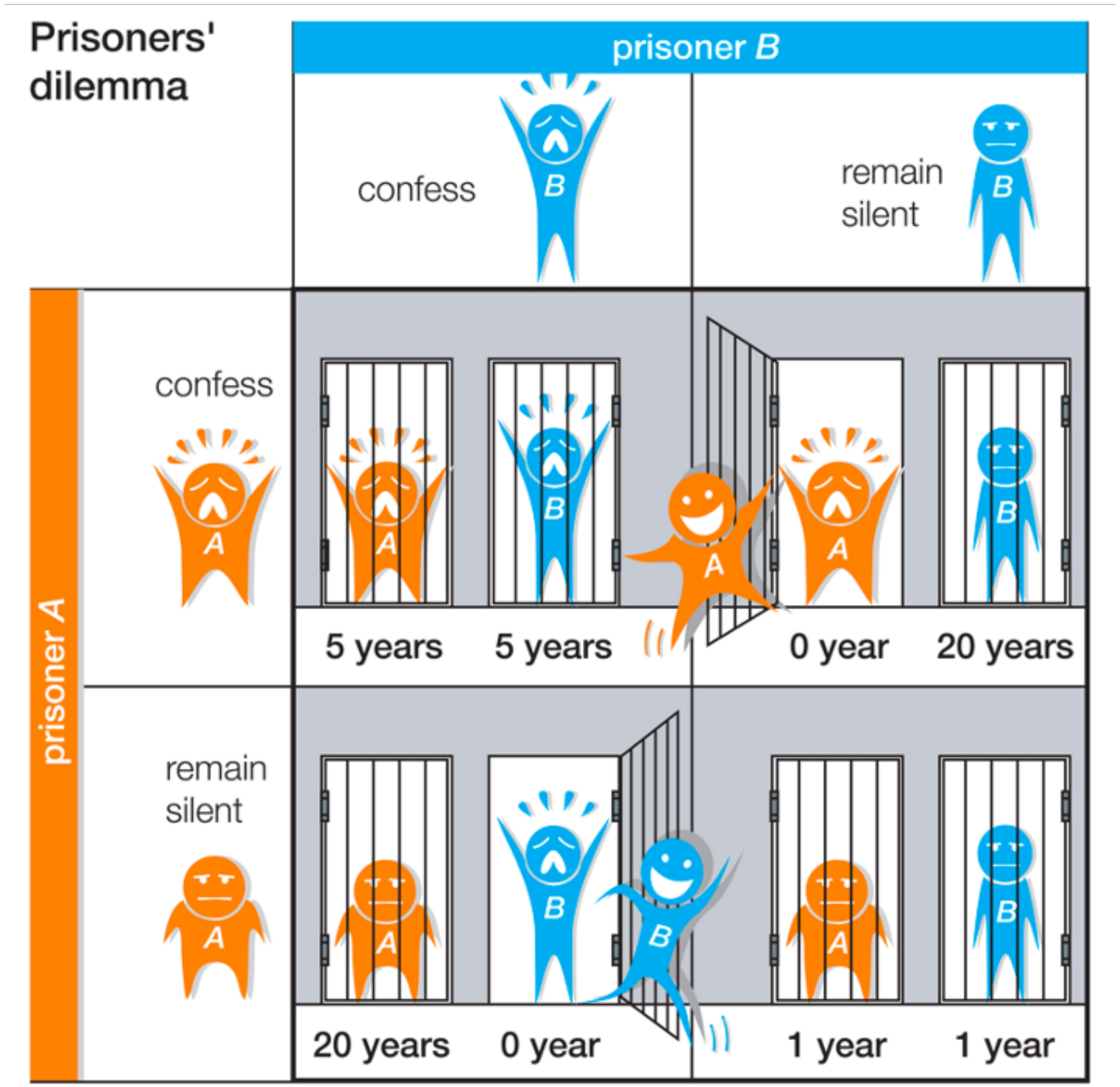
References: Camerer, C. F. (2011). *Behavioral game theory: Experiments in strategic interaction*. Princeton University Press.

# Solving prisoners’ dilemma






- Silent always does worse whatever the other player chooses, therefore it is a dominated strategy - there exists another strategy (in this case to confess) which always does better regardless of other players’ choices

# Dominance

- Strategy A (strongly) dominates B if the payoff from choosing A is higher than the payoff from B, regardless of what other players do.
- Strategy A weakly dominates B if A's payoffs are higher for some choices by others, and never lower.
- Are there any dominant and/or dominated strategies in this game?

| | | Player 2 | |
|---|---|---|---|
| | | *L* | *R* |
| Player 1 | *U* | 2, 3 | 5, 0 |
| | *D* | 1, 0 | 4, 3 |

# Dominance

- Strategy D is dominated by strategy U for Player 1.
  - D should never be played by a rational Player 1.
- Is there dominated strategy for player 2?
  - No.
  - Strategy L is better than R if player 1 selects U.
  - Strategy R is better than L is player 1 selects D.
  - L and R are undominated strategies for Player 2.

| | | Player 2 | |
|---|---|---|---|
| | | *L* | *R* |
| Player 1 | ***U*** | **2**, 3 | **5**, 0 |
| | ***D*** | 1, 0 | 4, 3 |

# Dominant Strategy Solution

- Are there any dominant strategies for Player 1?
- Are there any strictly dominant strategies for Player 1?
- Are there any dominant strategies for Player 2?
- Are there any strictly dominant strategies for Player 2?

| | | Player 2 | |
|---|---|---|---|
| | | *L* | *R* |
| Player 1 | *U* | 7, 3 | 5, 3 |
| | *D* | 7, 0 | 3, -1 |

# Dominant Strategy Solution

- When every player has a dominant strategy, the game has a dominant strategy solution
- In this example, (U, L) is a dominant strategy solution.

| | | Player 2 | |
|---|---|---|---|
| | | *L* | *R* |
| Player 1 | *U* | 7, 4 | 5, 3 |
| | *D* | 7, 0 | 3, -1 |

# Iterated elimination of strictly dominated strategies

- If a strategy is dominated, eliminate it
- The size and complexity of the game is reduced
- Eliminate any dominated strategies from the reduced game
- Continue doing so successively

| | | Player 2 | |
|---|---|---|---|
| | | *L* | *R* |
| Player 1 | *U* | 1, 1 | 0, 1 |
| | *M* | 0, 2 | 1, 0 |
| | *D* | 0, -1 | 0, 0 |

# Iterated elimination of strictly dominated strategies

| | | | Player 2 | |
|---|---|---|---|---|
| | | | *L* | ~~*R*~~ |
| | Player 1 | *U* | 1, 1 | ~~0, 1~~ |
| 3 | | ~~*M*~~ | ~~0, 2~~ | ~~1, 0~~ |
| 1 | | ~~*D*~~ | ~~0, -1~~ | ~~0, 0~~ |
| | | | | 2 |

| | | Player 2 | | |
|---|---|---|---|---|
| | | $L$ | $C$ | $R$ |
| Player 1 | $U$ | 8, 3 | 0, 4 | 4, 4 |
| | $M$ | 4, 2 | 1, 5 | 5, 3 |
| | $D$ | 3, 7 | 0, 1 | 2, 0 |

# Experiment: Guessing Game

- Each person will be asked to choose a number between (and including) 0 and 100 simultaneously. Communication is not allowed in this game.
- After the numbers are collected, the average of these numbers will be calculated.
- The person whose number is closest to, but not exceeding, 1/3 of the average (called the target number) will win.

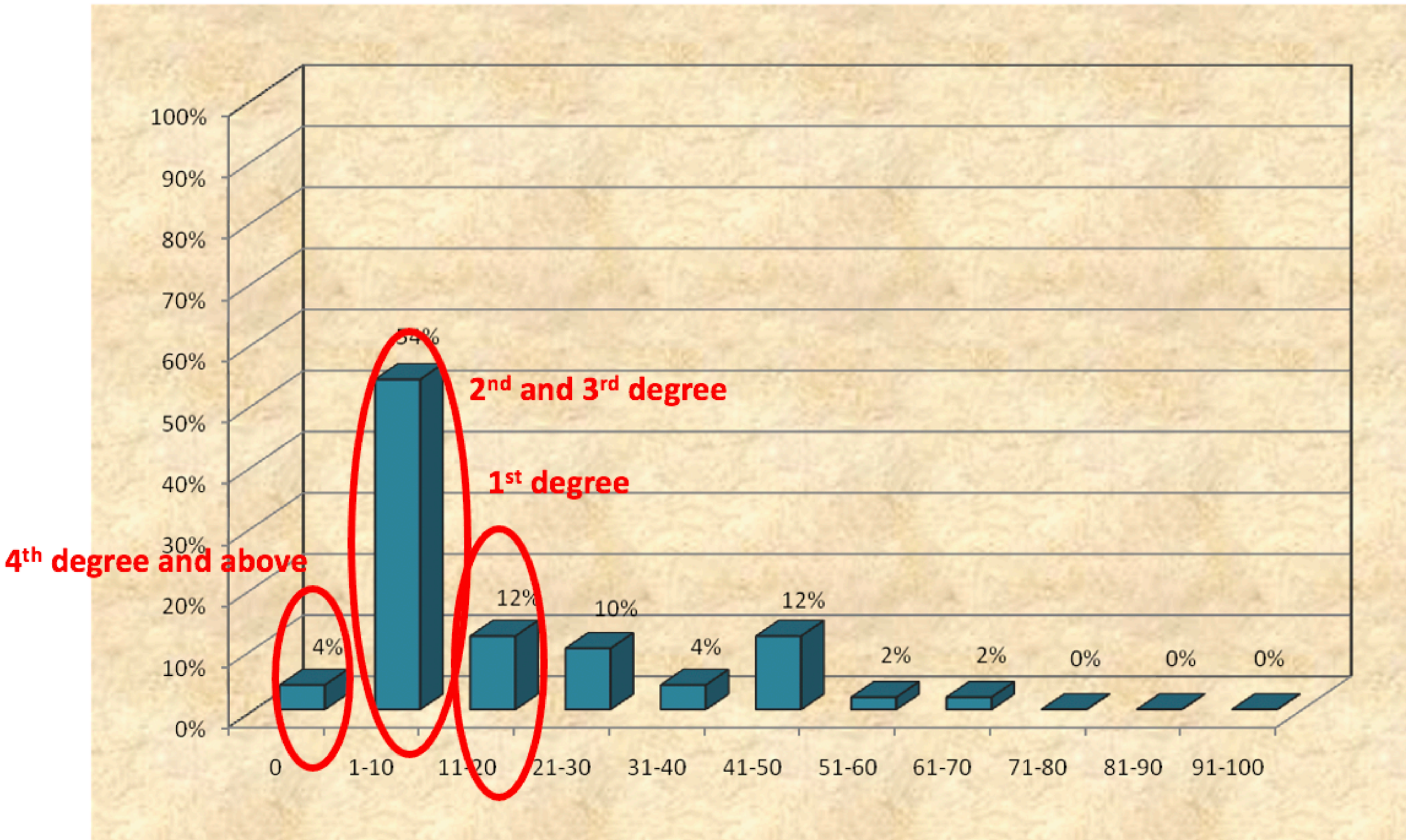
# Guessing game

- Guessing game = Beauty contest game
- Keynes described the action of rational actors in a market using an analogy based on a newspaper contest. Entrants are asked to choose a set of 6 faces from photographs that they find “most beautiful.” Those who picked the most popular face are eligible for a prize.
- Keynes (General Theory of Employment, Interest, and Money, 1936, p. 156): “It is not a case of choosing those which, to the best of one’s judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree, where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth, and higher degrees.”
- Keynes suggested that similar behavior is observed in the stock market. Shares are not priced based on what people think their fundamental value is, but rather on what they think everyone else thinks the value is and what they think about these beliefs, and so on.

# Guessing Game

- The guessing game can be used to distinguish whether people "practice the 4th, 5th, and higher degrees" of reasoning as Keynes wondered.
  - 1st: bid = 50; target = 50 * 1/3 = 17
  - 2nd: bid = 17; target = 17 * 1/3 = 6
  - 3rd: bid = 6; target = 6 * 1/3 = 2
  - 4th: bid = 2; target = 2 * 1/3 = 0.67
  - 5th: …
- In analytical game theory, players do not stop this iterated reasoning until they reach a best-response point.
- Unique (Nash) equilibrium = 0

# Guessing Game



# Guessing game

- Limited strategic thinking
- First stage – different levels of sophistication
- Most people – Level 1 or level 2
- Even if you know Nash equilibrium you usually don't want to play it
- Similar to bubbles in the stock market – everyone thinks he is one step ahead and continue playing/adjusting
- Nash equilibrium can be reached after few rounds – convergence

# New solution concept: Nash equilibrium

| | | Player 2 | | |
|---|---|---|---|---|
| | | L | C | R |
| | T | 0 , 4 | 4 , 0 | 5 , 3 |
| Player 1 | M | 4 , 0 | 0 , 4 | 5 , 3 |
| | B | 3 , 5 | 3 , 5 | 6 , 6 |

- The combination of strategies (B, R) has the following property:
- Player 1 CANNOT do better by choosing a strategy different from B, given that player 2 chooses R.
- Player 2 CANNOT do better by choosing a strategy different from R, given that player 1 chooses B.

# Nash Equilibrium: idea

- Nash equilibrium: A set of strategies, one for each player, such that each player's strategy is best for her, given that all other players are playing their corresponding strategies, or

- A stable situation that no player would like to deviate if others stick to it

- In a 2-player game, ( s1, s2 ) is a Nash equilibrium if and only if player 1's strategy s1 is her best response to player 2's strategy s2, and player 2's strategy s2 is her best response to player 1's strategy s1.

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| | | Prisoner 2 | |
|---|---|---|---|
| | | Quiet | Fink |
| Prisoner 1 | Quiet | -1 , -1 | -25, 0 |
| | Fink | 0 ,-25 | -20,-20 |

# Finding Nash Equilibrium

| | | Player 2 | | |
|---|---|---|---|---|
| | | L' | C' | R' |
| | T' | 0 , 4 | 4 , 0 | 3 , 3 |
| Player 1 | M' | 4 , 0 | 0 , 4 | 3 , 3 |
| | B' | 3 , 3 | 3 , 3 | 3.5 , 3.6 |

- If Player 2 chooses L' then Player 1's best strategy is…
- If Player 2 chooses C' then Player 1's best strategy is …
- If Player 2 chooses R' then Player 1's best strategy is …
- If Player 1 chooses T' then Player 2's best strategy is…
- If Player 1 chooses M' then Player 2's best strategy is…
- If Player 1 chooses B' then Player 2's best strategy is…

# Finding Nash Equilibrium

Player 2

| | | L' | C' | R' |
|---|---|---|---|---|
| Player 1 | T' | 0 , 4 | 4 , 0 | 3 , 3 |
| | M' | 4 , 0 | 0 , 4 | 3 , 3 |
| | B' | 3 , 3 | 3 , 3 | 3.5 , 3.6 |

- Best response: the best strategy one player can play, given the strategies chosen by all other players
- The Nash Equilibrium is (B', R')

# Matching coins

- What is Player 2’s best response to Player 1’s strategy Head?
- What is Player 2’s best response to Player 1’s strategy Tail?
- What is Player 1’s best response to Player 2’s strategy Head?
- What is Player 1’s best response to Player 2’s strategy Tail?

| | | Player 2 | |
|---|---|---|---|
| | | *H* | *T* |
| Player 1 | *H* | 1, -1 | -1, 1 |
| | *T* | -1, 1 | 1, -1 |

# Matching coins

- There are no combinations of best responses that match!

- Therefore, NO Nash equilibrium (in pure strategies. However, there exists an equilibrium in mixed strategies)

| | | Player 2 | |
|---|---|---|---|
| | | Head | Tail |
| Player 1 | Head | -1 , 1★ | ★1 , -1 |
| | Tail | ★1 , -1 | -1 , 1★ |

# Battle of genders

- What is Steve's best response to Rebecca's strategy Opera?
- What is Steve's best response to Rebecca's strategy Lakers?
- What is Rebecca's best response to Steve's strategy Opera?
- What is Rebecca's best response to Steve's strategy Lakers?

| | | Steve | |
|---|---|---|---|
| | | Opera | Lakers |
| Rebecca | Opera | 2 , 1 | 0 , 0 |
| | Lakers | 0 , 0 | 1 , 2 |

- There are two Nash Equilibria
- (Opera, Opera) is a Nash equilibrium
- (Lakers, Lakers) is a Nash equilibrium

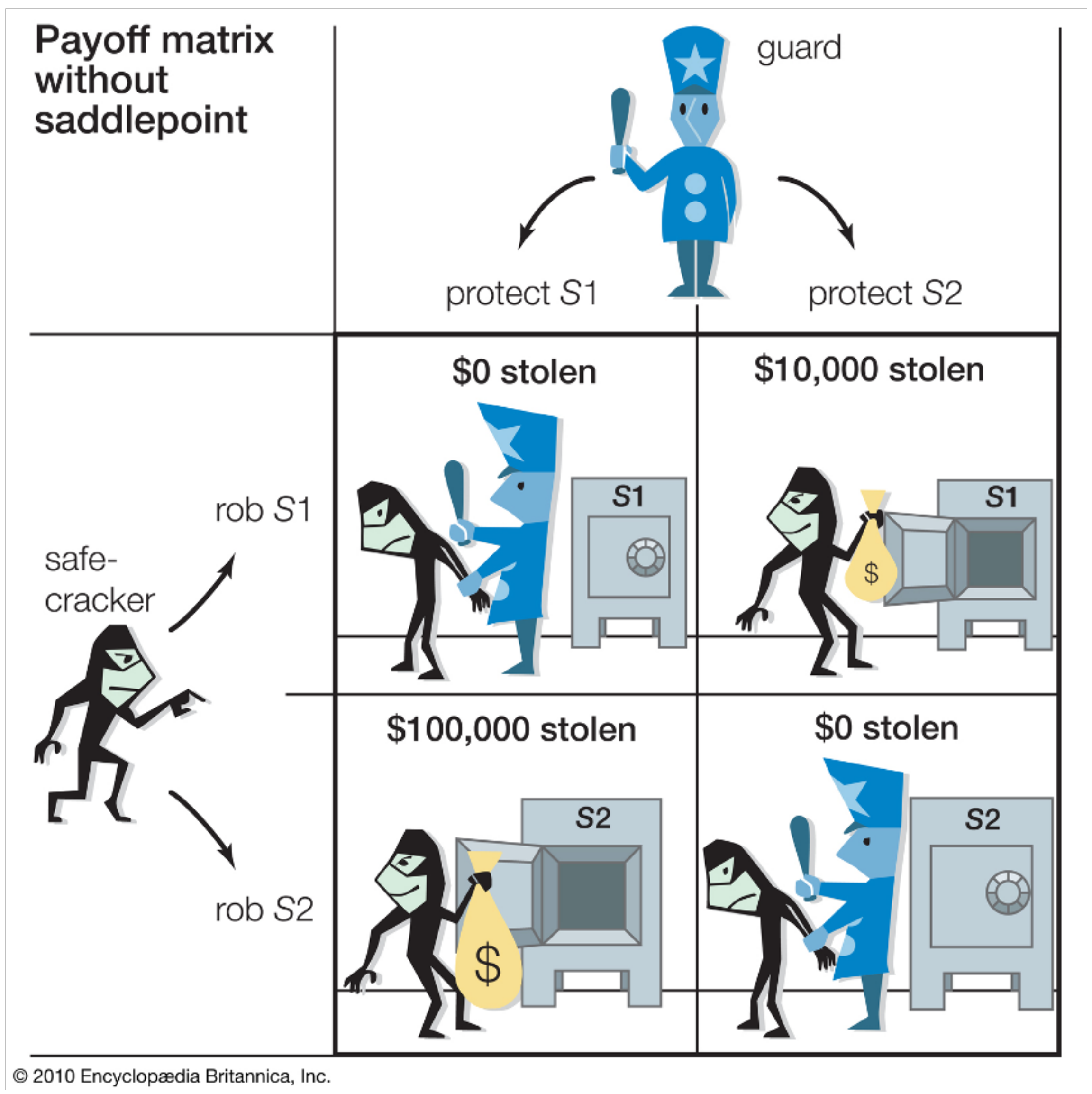
# Nash Equillibrium

- A Nash equilibrium prevails if players choose mutually best responses, i.e., each player chooses the strategy that maximizes his or her utility given the strategies played by the opponents.
- In other words, in a Nash equilibrium, no player has an incentive to choose another strategy than the one he or she is currently playing.
- Any game with finite player and strategy sets has an equilibrium at least in mixed strategies.
- A mixed strategy is a probability distribution over pure strategies.
- A mixed-strategy Nash equilibrium requires that the mixed strategies are mutual best responses.

# Mixed strategies

- In order to predict outcomes for games without (pure) Nash equilibria or with multiple equilibria, we need an extension of the concepts of strategies and equilibria.
- Randomization of moves and mixed strategies
    - The need for randomizing moves in the play of a game usually arises when one player prefers a coincidence of actions, while his rival prefers to avoid it.
    - Each player would like to outguess the other.
    - Examples: the matching pennies game, tennis matches, soccer penalty kicks
    - In all these games, players want to take advantage of the element of surprise. They want to be unpredictable. The skill to being unpredictable requires understanding and being able to find the mixed-strategy equilibria of these games. Mixed strategies are not intuitive.

# Mixed strategies




- If the guard protects S1 with probability 1/11 and S2 with probability 10/11, he will lose, on average, no more than about $9,091 whatever the safecracker does.
- Using the same kind of argument, it can be shown that the safecracker will get an average of at least $9,091 if he tries to steal from S1 with probability 10/11 and from S2 with probability 1/11.
- The safecracker and the guard give away nothing if they announce the probabilities with which they will randomly choose their respective strategies.
- On the other hand, if they make themselves predictable by exhibiting any kind of pattern in their choices, this information can be exploited by the other player.

# Mixed strategies

- Players want to be “unpredictable” (for instance, a tennis player doesn’t want to be predictable about whether she serves the ball left or right). Being unpredictable requires playing a mixed strategy. In any mixed-strategy equilibrium players will choose probability distributions such that their opponent will be indifferent in choosing his or her pure strategies.
- Yet, behaviorally, there are at least three problems.
  - First, in equilibrium, players have to accurately guess the exact probabilities with which the opponents will play their mixed strategies.
  - Second, players should really randomize their choices. However, it is well known from psychological research that people are not very good in producing random sequences
  - Third, learning is difficult, because in equilibrium people are indifferent between their choices. This implies that there are no positive incentives for playing a particular strategy.
- Yet, the degree to which human players display behavior that is consistent with the mixed-equilibrium prediction is an empirical question
- New experiments report results that are favorable for mixed-strategy equilibrium in the sense that the observed frequencies are close to the theoretical frequencies. These results are quite surprising and good news for the mixed-equilibrium prediction, given that there are sound psychological reasons to assume that the concept is behaviourally rather demanding.

# Do Soccer Players Flip Coins?

- Penalty kicks
  - Kicker's strategy space: {L,M,R}
  - Goalie's strategy space: {L,M,R}
- Simultaneous move game? (125mph, 0.2 seconds reaction time)
- What's the Nash equilibrium?
- What do players do in reality?

## Penalty Kicks

Chiappori, Levitt, and Groseclose (2002)

- 459 kicks in French and Italian first leagues
- 162 kickers, 88 goalies

TABLE 3—OBSERVED MATRIX OF SHOTS TAKEN

| | Kicker | | | |
|---|---|---|---|---|
| Goalie | Left | Middle | Right | Total |
| Left | 117 | 48 | 95 | 260 |
| Middle | 4 | 3 | 4 | 11 |
| Right | 85 | 28 | 75 | 188 |
| Total | 206 | 79 | 174 | 459 |

*Notes:* The sample includes all French first-league penalty kicks from 1997–1999 and all Italian first-league kicks (1997–2000). For shots involving left-footed kickers, the directions have been reversed so that shooting left corresponds to the "natural" side for all kickers.

# Hawk vs. Dove (Chicken)

- The story is that two teenagers drive home on a narrow road with their bikes, and in opposite directions.
- None of them wants to go out of the way
  - whoever 'chickens' out loses his pride, while the tough guy wins.
  - if both stay tough, then they break their bones!!
  - if both go out of the way, then only their pride is damaged slightly.

| | | Player 2: Tough | Player 2: Chicken |
|---|---|---|---|
| Player 1 | Tough | -10,-10 | 1,-1 |
| | Chicken | -1,1 | 0,0 |

# Hawk vs. Dove

- The hawk-dove model is an evolutionary game theoretical model developed by John Maynard Smith (1982) depicting the fundamental conflict between prosocial (altruism and cooperation) and antisocial behavior (selfishness).

- The model describes the contest between two fundamentally different behavioral strategies, hawks (selfishness) and doves (prosociality), when competing over a shared resource. This contest reveals the evolutionary paradox of prosocial behavior (i.e., if natural selection is based on competition, then prosocial traits should not evolve).

- The hawk-dove model provides a simplistic framework to investigate the conditions that favor the evolution of prosocial behavior. Overall, hawks outcompete doves within groups, but a group of doves outcompete a group of hawks. For either hawks or doves to evolve, the balance of selection within and between groups must tip in their respective favor.

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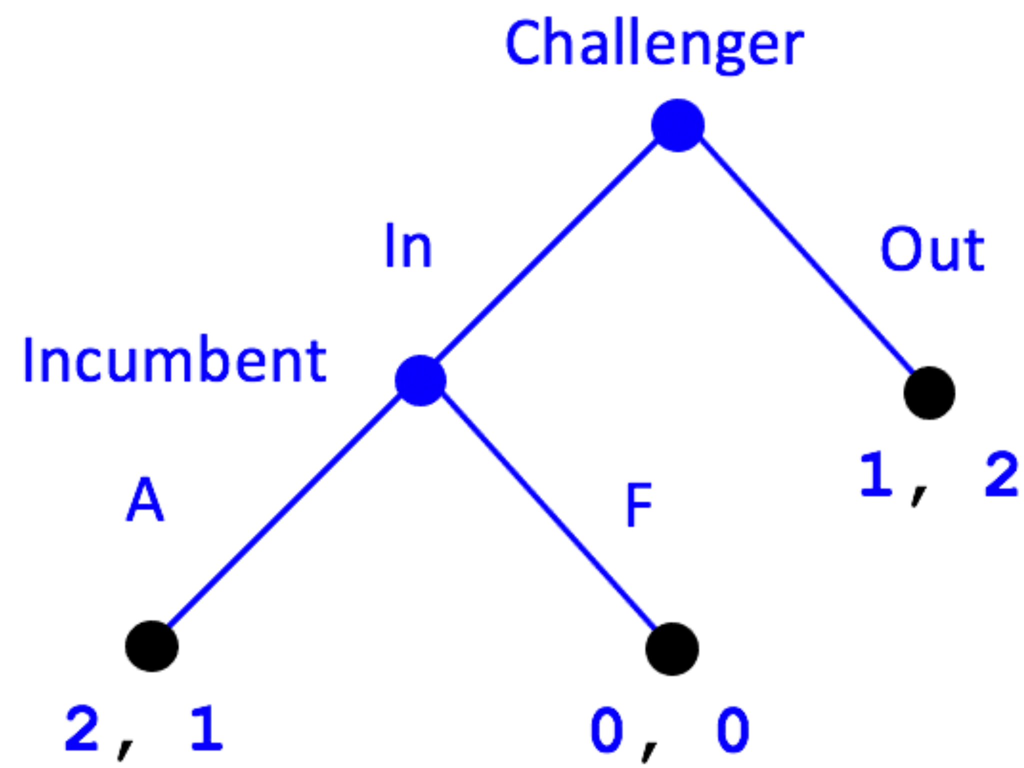
# Sequential (dynamic) Games: Market Entry

- An incumbent monopolist faces the possibility of entry by a challenger.
- The challenger may choose to enter or stay out.
- If the challenger enters, the incumbent can choose either to accommodate or to fight.
- The payoffs are common knowledge.
- The first number is the payoff of the challenger. The second number is the payoff of the incumbent.



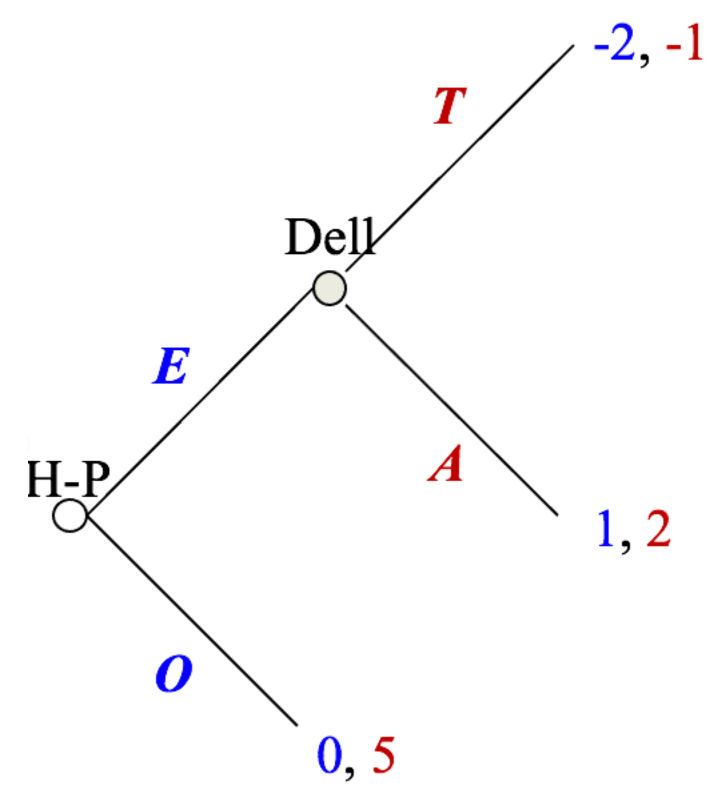
# Sequential (dynamic) Games: Market Entry

- Suppose H-P is (Challenger) debating whether or not to enter a new market, where the market is dominated by its rival, Dell (Incumbent).

- Both firms' profitability depends on how Dell is going to react to H-P coming into the market.

  - 1. Dell mounts a big advertising campaign to secure its market share (playing "tough") AND both firms lose money.

  - 2. Dell does not mount such a tough counterattack.



# Sequential (dynamic) Games: Market Entry

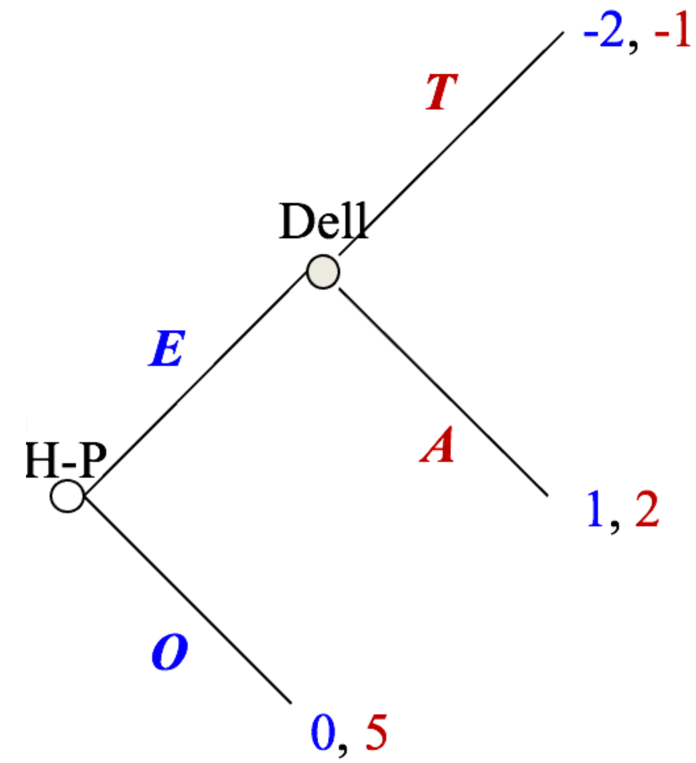
- Which of the Nash equilibria is a reasonable play? (O, T)
- Is H-P's O the best response to Dell's T? Yes.
- But is Dell's T a credible threat for H-P? No.
- By entering the market, H-P knows that Dell would rather accommodate.
- H-P may not find a T response from Dell being credible at all.
- (E, A): the only reasonable N. E

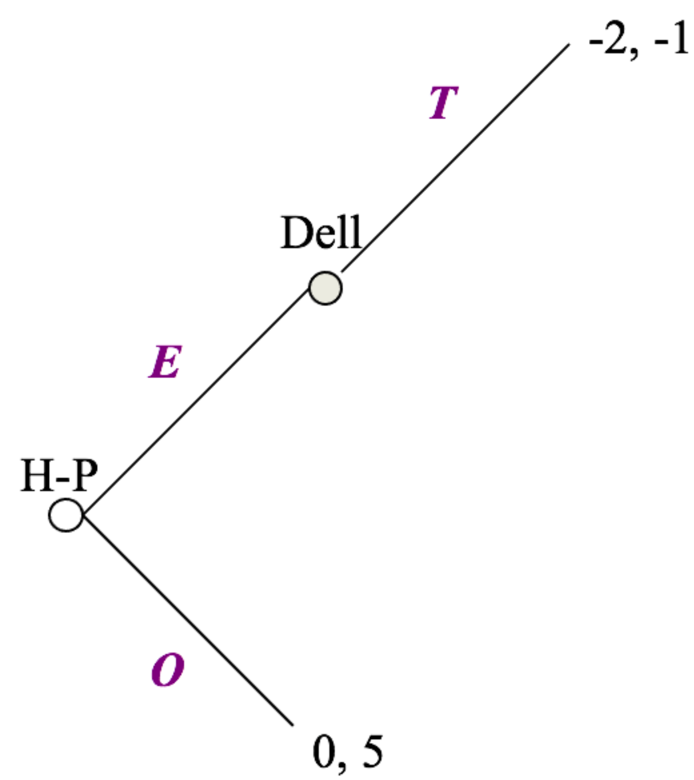
| | | Dell | |
|---|---|---|---|
| | | *T* | *A* |
| H-P | *E* | -2, -1 | 1, 2 |
| | *O* | 0, 5 | 0, 5 |

| | | Dell | |
|---|---|---|---|
| | | *T* | *A* |
| H-P | *E* | -2, -1 | 1, 2 |
| | *O* | ~~0, 5~~ | 0, 5 |

# The Power of Commitment

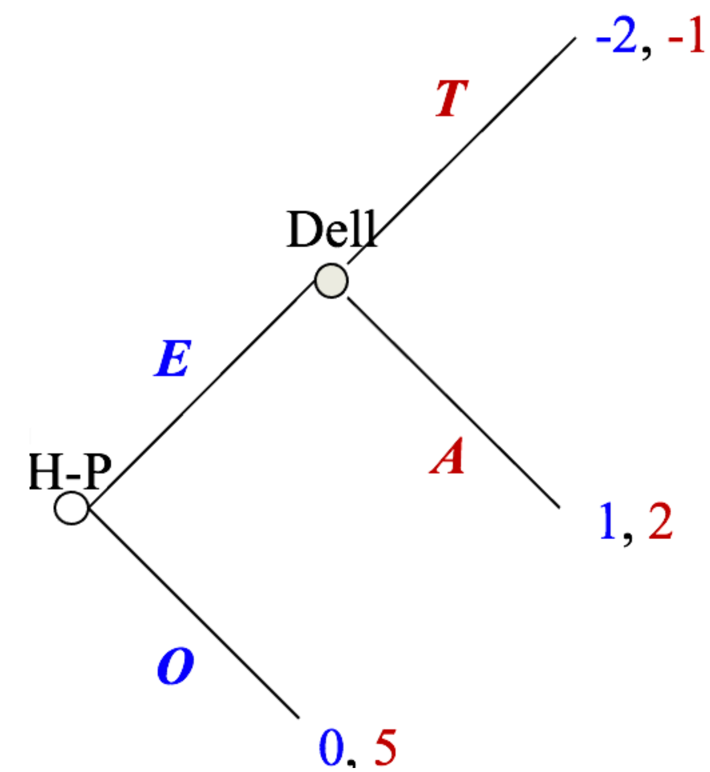
- The above example embodies the assumption that, at the beginning of the game, the incumbent (Dell) cannot commit to fight if the challenger enters.
- It is free to choose either T or A in this event.
- If the incumbent could commit to fight in the event of entry, then the game would be completely different.
- If Dell commits to fight after H-P enters the market, the rational thing for H-P to do is to stay out. Less (choices) can mean more (equilibrium payoff)!

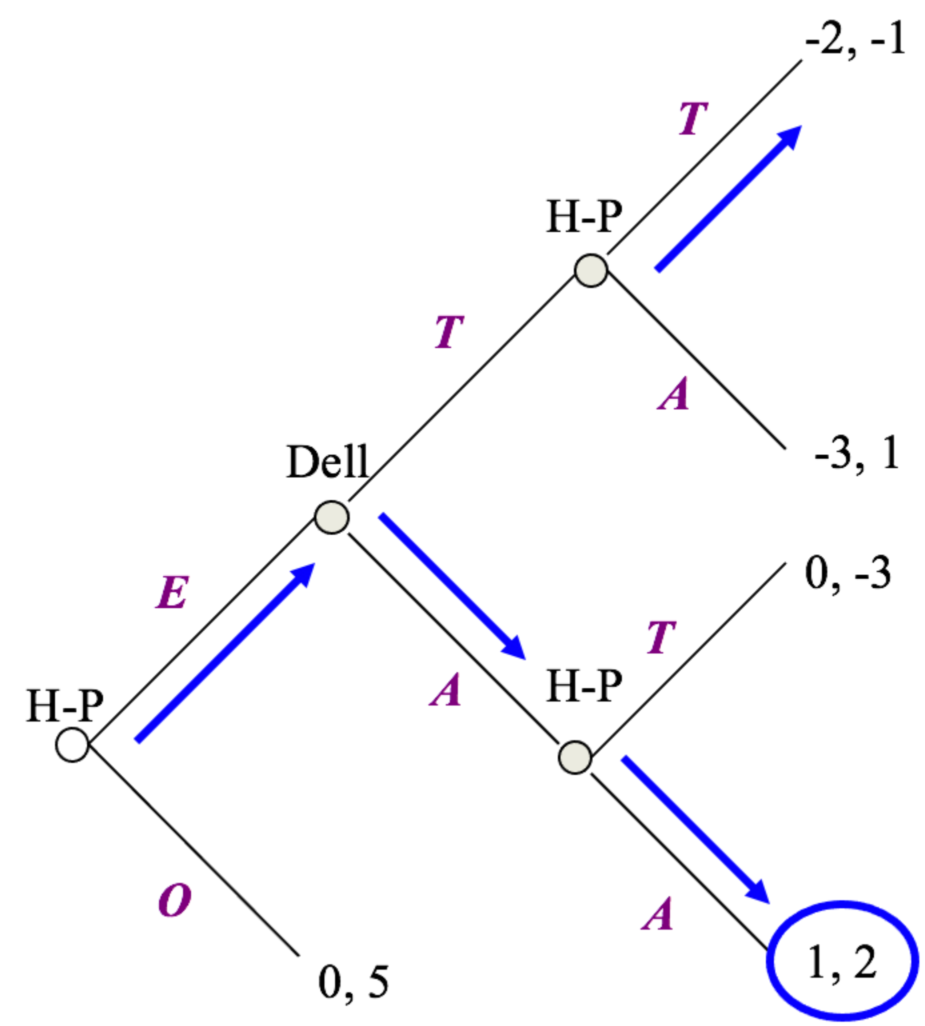


# Backward Induction

- Solve the game using backward induction to eliminate non-credible threats.
- Each player makes optimal decisions at every stage.
- Subgame Perfect NE : (ETA, A) with the payoff of (1, 2)



# Centipede game

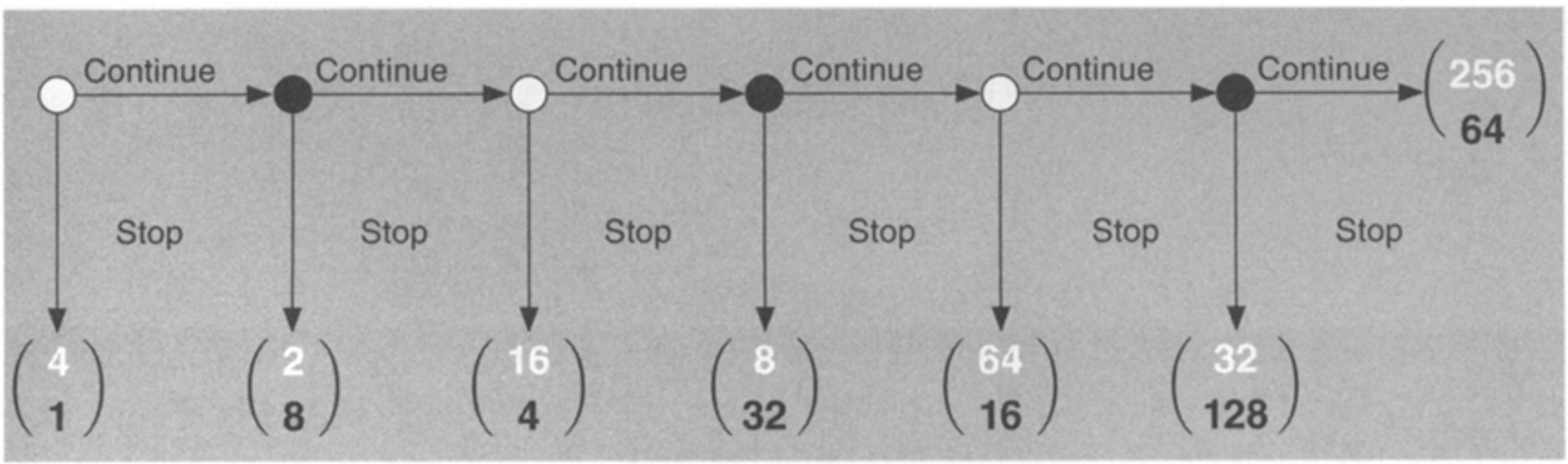



FIGURE 1. A CENTIPEDE GAME

# Centipede game

- Empirical evidence over several decades from a number of experimental lab studies documents systematic departures from the backward induction outcome in various games. These studies often conjecture that various forms of social preferences, limited cognition or simply failures of backward induction reasoning could explain why the equilibrium outcome is rarely observed in the lab.

- Palacios-Huerta and Volij (2009) differs from those studies in the subject pool considered. Instead of students, the study looks at subjects who are likely to be characterized by a high degree of rationality and devote a large part of their life to finding optimal strategies using backward induction reasoning: chess players. Interest in these subjects stems from the fact it is safe to say that it is common knowledge that chess players are highly familiar with backward induction reasoning. This makes them ideal subjects for studying the extent to which knowledge of an opponent's rationality is a key determinant of the predictive power of subgame-perfect equilibrium.

- We consider the classic centipede game, which features a counterintuitive equilibrium outcome and show in both the field and the lab, when chess players play against each other the outcome is very close to the subgame-perfect equilibrium prediction. In various lab experiments, we also find that more than 70 percent of games end at the first node in the game, and that every chess player converges fully to equilibrium play by the fifth repetition.

- Also, when students play against chess players the outcome is much closer to the subgame-perfect equilibrium than when students play against students. These results were later confirmed in Gil and Prowse (2016) who find that more cognitively able students converge more frequently to equilibrium play in repeated strategic interactions and respond positively to the cognitive ability of their opponents. It means that the predictive power of subgame-perfect equilibrium hinges mainly on knowledge of players' rationality and not on altruism or social preferences.

# First mover advantage

- Football penalty shoot-outs are not a 50-50 lottery. It is more like a 60-40 lottery, i.e. the order of play is a strongly significant determinant of winning and there is a sizeable advantage for the team that is first to kick. That team is randomly given a greater chance to be leading in the tournament, and this leading-lagging asymmetry between the two teams appears to cause psychological differences that impact performance. Interestingly, individuals are typically aware of this effect and respond rationally to it by typically choosing to kick first when given the chance.

- Similar results apply to chess. Observed frequencies to win are again about 60-40 in favor of the player drawing the white pieces in the first game.

- Interestingly, the opposite holds true for shoot-out in ice hockey, where the scoring rate of a penalty is low (about 33 percent) rather than high. Here the goalie can be viewed as "taking the penalty" (he 'scores' when he saves the penalty). And indeed, there is an advantage for the team whose goalie goes first in the sequence.

- To reconcile the unfairness, instead of traditional ABAB sequence, one can use ABBA sequence as is in place for example in tennis tie-breaks, where we do not observe first (and also not second) mover advantage. Some football tournaments in England actually used ABBA sequence. In 36 such shoot-outs, Team A won 18 times, and Team B 18 times. That is, in these trials ABBA worked in creating a more fair and balanced outcome, bringing the frequencies with which the A and B teams win closer to 50-50 than to 60-40. In fact, in this sample they are exactly 50-50: perfectly ex post fair.

- https://www.youtube.com/watch?v=S0qjK3TWZE8