Andrew, Beth, and Cathy live in Lindhville. Andrew's demand for bike paths, a public good, is given by Q = 12 – 2P. Beth's demand is Q = 18 – P, and Cathy's is Q = 8 – P/3. The marginal cost of building a bike path is MC = 21. The town government decides to use the following procedure for deciding how many paths to build. It asks each resident how many paths they want, and it builds the largest number asked for by any resident. To pay for these paths, it then taxes Andrew, Beth, and Cathy the prices a, b, and c per path, respectively, where a + b + c = MC. (The residents know these tax rates before stating how many paths they want.)

a. If the taxes are set so that each resident shares the cost evenly (a = b = c), how many paths will get built?

When taxes are set at a = b = c = MC/3 = 7, each resident faces an individual marginal cost of 7 per bike path. At this marginal cost, Andrew wants no bike paths, Beth wants 11, and Cathy wants 2.67. The government therefore builds 11 paths.

b. Show that the government can achieve the social optimum by setting the correct tax prices a, b, and c. What prices should it set?

The social optimum can be computed by reexpressing the demand curves for the three residents as P = 6 – Q/2, P = 18 – Q, and P = 24 – 3Q, respectively, and summing them to get marginal social benefit MSB = 48 – 4.5 Q. Setting MSB = MC and solving for Q gives Q = 6. We need to tax prices so that nobody will want more than 6 units (and someone will want exactly 6 units). Looking at Andrew's inverted demand curve, we see that he will want exactly 6 units at a = 3 (since then a = 6 – 6/2). Beth will want exactly 6 units at b = 12. And at c = 6, Cathy will want exactly 6 units. Since 3 + 12 + 6 = 21, these tax rates are just enough to cover MC, and the social optimum is achieved. Note that with this tax system in place, the three residents are unanimous in the number of bike paths they desire.