Firms A and B each produce 80 units of pollution. The federal government wants to reduce pollution levels. The marginal costs associated with pollution reduction are MCA = 50 + 3QA for firm A and MCB = 20 + 6QB for firm B, where QA and QB are the quantities of pollution reduced by each firm. Society's marginal benefit from pollution reduction is given by MB = 590 – 3Qtot, where Qtot is the total reduction in pollution.

a. What is the socially optimal level of each firm's pollution reduction?

The social optimum requires MB = MCA= MCB. Setting MCA= MCB and solving for QA in terms of QB,

50 + 3QA = 20 + 6QB, or QA = 2QB – 10.

Setting MCB = MB, plugging in for QA in terms of QB, and solving for QB,

20 + 6QB = 590 – 3Qtot = 590 – 3(QA+ QB) = 590 – 3((2QB – 10)+ QB)

20 + 6QB = 620 – 9QB

15QB = 600QB = 40.

Plugging back in for QA gives QA = 70.

b. How much total pollution is there in the social optimum?

There was a total of 160 units before, and the social optimum has 40 + 70 = 110 units of pollution reduction, so the social optimum has 50 units of pollution.

c. Explain why it is inefficient to give each firm an equal number of pollution permits (if they are not allowed to trade them).

It would be inefficient for each firm to be give 25 units of pollution permits. Then each firm would have to reduce pollution by 55 units. The cost to firm B of the 55th unit of reduction was 350. The cost to firm A of reducing pollution by an additional unit (the 56th unit) would be 218. Hence, efficiency could be improved by having firm B produce 1 unit more pollution and having A produce 1 unit less: there would be no change in total pollution, but the total cost of abatement would go down by 132.

d. Explain how the social optimum can be achieved if firms are given equal numbers of pollution permits but are allowed to trade them.

Starting from the situation where each firm has 25 units of pollution, note that firm A would be willing to reduce pollution by another unit if it was paid 218 or more. Firm B would be willing to pay up to 350 to have another pollution permit. If permits were tradeable, both firms would benefit by

firm A selling a pollution permit to firm B at some intermediate price. They would keep making profitable sales of this sort until the social optimum was reached and the cost of an additional unit of pollution reduction was the same for both firms.

e. Can the social optimum be achieved using a tax on pollution?

A tax could be used to achieve the same outcome: setting a tax of 260 would lead firm A to reduce pollution until 50 + 3QA = 260, yielding QA = 70. It would lead firm B to reduce pollution until 20 + 6QB = 260, yielding QB = 40. Hence, the tax would achieve the social optimum.